Math 115

Fall 2017

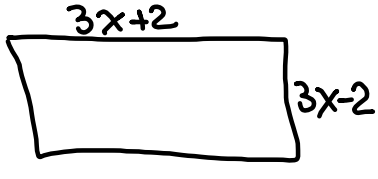
Lecture 20

Class Quiz

1) Simplify: $(-4x^5)^3 = (-4)^3(x^5)^3 = \boxed{-64x^{15}}$

2) $(x^{-4})^5 \cdot x^8 = x^{-20} \cdot x^8 = x^{-12} = \frac{1}{x^{12}}$

3) Find the area of



$A = (3x+2)(3x-2)$

$= 9x^2 - 6x + 6x - 4 \Rightarrow \boxed{A = 9x^2 - 4}$

Ch. 5 : Factoring

Writing a Polynomial as a product of other polynomials

1) GCF : Greatest Common Factor

$15x + 25 = \boxed{5} \cdot 3x + \boxed{5} \cdot 5$

$= \boxed{5(3x+5)}$

Factor out the GCF

1) $24x^2 - 16x$

$= \boxed{8x} \cdot 3x - \boxed{8x} \cdot 2$

$= \boxed{8x(3x-2)}$

2) $45x^3y^2 - 18xy^3$

$= \boxed{3} \cdot \boxed{3} \cdot 5 \cdot \boxed{x} \cdot x \cdot x \cdot \boxed{y} \cdot \boxed{y} - 2 \cdot \boxed{3} \cdot \boxed{3} \cdot \boxed{x} \cdot \boxed{y} \cdot \boxed{y} \cdot y$

$= \boxed{9xy^2(5x^2 - 2y)}$

3) $17x(2x-5)-4(2x-5)$

$=\boxed{(2x-5)(17x-4)}$

Factor by grouping (4 terms or more)

$\boxed{2x^3+5x^2}\ \boxed{+4x+10}$

$=x^2(2x+5)+2(2x+5)$

$=(2x+5)(x^2+2)$

$7x^3-8x^2+21x-24$

$=x^2(7x-8)+3(7x-8)$

$=\boxed{(7x-8)(x^2+3)}$

$4x^4+9x^3-8x^2-18x$

$=x\left[\underbrace{4x^3+9x^2}_{x^2(4x+9)}\underbrace{-8x-18}_{-2(4x+9)}\right]=\boxed{x(4x+9)(x^2-2)}$

Factor:

1) $55x^4 - 22x^3 = \boxed{11x^3(5x - 2)}$

2) $3x^2(2x-5) - 4x(2x-5) + 10(2x-5)$

$= (2x-5)(3x^2 - 4x + 10)$

3) $3x^3 + 4x^2 - 6x - 8$

$= x^2(3x+4) - 2(3x+4) = (3x+4)(x^2-2)$

Factoring Trinomials in the form of

$$ax^2 + bx + c \quad ; a \neq 0$$

$3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2$

$P = 6$

$S = 5$

6

$1, 6$

$2, 3$

$= 3x(x+1) + 2(x+1)$

$= \boxed{(x+1)(3x+2)}$

$3x^2 + 2x - 5 = 3x^2 - 3x + 5x - 5$

$P = -15$

$S = 2$

-15

$-1, 15$

$-3, 5$

$= 3x(x-1) + 5(x-1)$

$= (x-1)(3x+5)$

1) $x^2 + 13x + 36 = x^2 + 9x + 4x + 36$

$P = 36$

$S = 13$

36

$9 \& 4$

$= x(x+9) + 4(x+9)$

$= (x+9)(x+4)$

2) $x^2 - 5x - 36 = x^2 - 9x + 4x - 36$

$P = -36$

$S = -5$

-36

$-9 \& 4$

$= x(x-9) + 4(x-9)$

$= (x-9)(x+4)$

Factor $2x^2 - 13x - 15$

$1, -30$

$2, -15$

$3, -10$

$5, -6$

$P = -30$

$S = -13$

-30

$= 2x^2 + 2x - 15x - 15$

$= 2x(x+1) - 15(x+1)$

$= \boxed{(x+1)(2x-15)}$

$5x^2 + 6x - 8$

$-1, 40$

$-2, 20$

$-4, 10$

$-5, 8$

$P = -40$

$S = 6$

-40

$= 5x^2 - 4x + 10x - 8$

$= x(5x-4) + 2(5x-4)$

$= (5x-4)(x+2)$

$4x^2 \quad -12x \quad +9$

$-1, -36$

$-2, -18$

$-3, -12$

$-4, -9$

$-6, -6$

36

$P = 36$

$S = -12$

$(+)(+) = +$

$(-)(-) = +$

$= 4x^2 - 6x - 6x + 9$

$= 2x(2x-3) - 3(2x-3)$

$= (2x-3)(2x-3)$

$= \boxed{(2x-3)^2}$

$5x^2 + 10x - 3$

$-1, 15$

$-3, 5$

-15

$P = -15$

$S = 10$

Not possible $\Rightarrow$ Non-factorable

$\Rightarrow$ Prime

Special Factoring:

Two terms

1) $A^2 + B^2 \Rightarrow$ Prime

2) $A^2 - B^2 = (A+B)(A-B)$

3) $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

4) $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$x^2 + 25 = x^2 + 5^2$ Prime

$x^2 - 49 = x^2 - 7^2 = (x+7)(x-7)$

$64x^2 - 25 = (8x)^2 - (5)^2$

$= (8x+5)(8x-5)$

$49x^2 + 100 = (7x)^2 + (10)^2$

$=$ Prime

$$121x^2 - 36y^2 = (11x)^2 - (6y)^2$$

$$= (11x + 6y)(11x - 6y)$$

$$64x^2 - 49y^2 = (8x)^2 - (7y)^2$$

$$= (8x + 7y)(8x - 7y)$$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$x^3 + 8 =$$

$$x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$$

$$x^3 + 125 =$$

$$x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$$

$8x^3 + 27 =$

$(2x)^3 + (3)^3 = (2x+3)(4x^2 - 6x + 9)$

$64x^3 + 125y^3 =$

$(4x)^3 + (5y)^3 = \boxed{(4x+5y)(16x^2 - 20xy + 25y^2)}$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$x^3 - 1000 =$

$x^3 - 10^3 = (x - 10)(x^2 + 10x + 100)$

$8x^3 - 343 =$

$(2x)^3 - (7)^3 = (2x-7)(4x^2 + 14x + 49)$

$125x^3 - 27y^3 =$

$(5x)^3 - (3y)^3 = (5x-3y)(25x^2 + 15xy + 9y^2)$

$x^8 - 256 = (x^4)^2 - (16)^2$

$= (x^4 + 16)(x^4 - 16)$

$= (x^4 + 16)(x^2 + 4)(x^2 - 4)$

$= \boxed{(x^4 + 16)(x^2 + 4)(x + 2)(x - 2)}$

Special Factoring with Trinomials

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Perfect-Square Trinomial

$x^2 + 20x + 100 = (x + 10)^2$ ✓

$2 \cdot x \cdot 10$

$25x^2 + 40x + 16 = (5x + 4)^2$ ✓

$2 \cdot 5x \cdot 4$

$$49x^2 - 126xy + 81y^2$$

$$= (7x - 9y)^2 \checkmark$$

$$2 \cdot 7x \cdot 9y$$

$$625x^2 - 100xy + 4y^2$$

$$= (25x - 2y)^2 \checkmark \qquad 2 \cdot 25x \cdot 2y$$

Divide $\dfrac{32x^6 - 16x^4 - 8x^2}{4x^3}$

$$= \frac{32x^6}{4x^3} - \frac{16x^4}{4x^3} - \frac{8x^2}{4x^3}$$

$$= \boxed{8x^3 - 4x - \frac{2}{x}}$$ Not a Polynomial.

Divide $\dfrac{x^4 - 5x^2 - 36}{x^2 - 9}$

$x^2 \square = x^4$

$x^2 \square = 4x^2$

$x^2 + 4$

$$x^2 - 9 \overline{)\, x^4 + 0x^3 - 5x^2 + 0x - 36}$$

$x^2 + 4$

$-(x^4 \quad -9x^2)$

$4x^2 + 0x - 36$

$-(4x^2 \quad -36)$

0

Area of a rectangle is $5x^3 - 2x^2 + 4x - 7$.

the width is $x - 1$.

Find its length.

$A = LW$

$\dfrac{A}{W} = L$

$x \square = 5x^3$

$x \square = 3x^2$

$x \square = 7x$

$5x^2 + 3x + 7$

$$x - 1 \overline{)\, 5x^3 - 2x^2 + 4x - 7}$$

$5x^2 + 3x + 7$

$-(5x^3 - 5x^2 \quad)$

$3x^2 + 4x - 7$

$-(3x^2 - 3x \quad)$

$7x - 7$

$-(7x - 7)$

0